to be useful in photochemical work. The values of the function e^{-x} are tabulated in thirteen pages from $x=0$ to $x=10$, and fifty-six pages are assigned to tables by Dr. N. Rosanow showing the reciprocal of the wave length and the frequency for every Ångström unit from λ 2000 to λ 8000. H. S. A.

The Economics of Everyday Life. Part i. By T. H. Penson. Pp. xiv+174. (Cambridge University Press, 1913.) Price 3*s*. net.

IT is surprising how difficult it apparently is to write a good short text-book of economics, but Mr. Penson has been eminently successful in doing so. He has fully grasped the fact that the first need for such a book is to be simple and elementary as well as short. Where possible, he rightly prefers the ordinary terms of everyday use to the technical phrases of economics. For instance, instead of production, exchange and distribution, he talks of the "source of income," "buying and selling," and the "individual income." These, in my opinion, are far more intelligible to the beginner. Moreover, his definitions are nearly always both clear and adequate, those of demand and supply affording a good example.

The method of treatment follows, on the whole, that of the modern school, of which Prof. Marshall may be regarded as the head, and exchange is treated before, and not after, distribution. The subjects of consumption, taxation, trade unions and cooperative societies are left to the second part of this book, which has yet to be published.

The present volume clearly marks Mr. Penson as possessing great capacity as a teacher. He chooses wisely not only his terms, but the subjects of which he treats. Omitting nothing that is essential, he has avoided thorny and difficult subjects likely to confuse the beginner. His definitions, too, are both concise and complete. A new and valuable feature of the book is found in the simple tables and diagrams by which the argument is rendered easy to understand, but mathematical methods are rigidly, and in such a book rightly, avoided. Occasionally, however, the author treats unimportant matters somewhat too fully. Usually he is neither too long nor too short, but, like Sidney Godolphin, "is never in the way, and never out of it." N. B. DEARLE.

Dent's Practical Notebooks of Regional Geography. By H. Piggott and R. J. Finch. Book i., The Americas. Pp. 64. (London: J. M. Dent and Sons, Ltd., 1913.) Price 6*d*. net.

IF every geography teacher set the same practical exercises, this conveniently arranged notebook would have a wide circulation; but naturally a teacher's exercises should reflect his own individuality. The little book may be commended, however, as affording a good example of the way in which pupils can be led to acquire an intelligent knowledge of geography as the result of their own activities.

LETTERS TO THE EDITOR.

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts intended for this or any other part of* NATURE. *No notice is taken of anonymous communications.*]

An Application of Mathematics to Law.

I HAVE attempted to apply mathematical symbolism to some of the difficult problems of patent law. The question to be decided by the Court in a patent law suit is usually this: assuming that the alleged invention deals with "a manner of manufacture" (*i.e.* is, or yields, something concrete), was there ingenuity and utility in the step from what was already known? Ingenuity means inventive or creative ingenuity as apart from the normal dexterity of the craftsman, which of itself is insufficient to support a patent, as otherwise patents would unduly hamper industry. It will be seen at once that it is a most subtle question for any court to determine whether a given act, the selection of one out of many alternatives, the assemblage of various old elements, the adaptation of old elements to new uses—whether such an act is one which calls for ingenuity as apart from the expected skill of the craftsman.

To express the problem symbolically I will start from an admirable dictum of Lord Justice Fletcher Moulton (Hickton Pat. Syn. **v.** Patents Improvements). He stated that invention might reside in the idea, or in the way of carrying it out, or in both; but if there was invention in the idea plus the way of carrying it out, then there was good subject-matter for a patent. I express this by representing any idea as a functional operator, and the way of carrying it out (*i.e.* the concrete materials adopted) as a variable. Calling result I:

$$I=f(x).$$

Here I represents what the Germans call the "technical effect" of the invention, or what Frost calls the manufacturing "art," and we see at once that a patent cannot be obtained for a mere principle or idea (f, which is not concrete) unless some way of carrying it out (x) is also given. But the invention may reside either in f or in x.

Let us express in general terms a manufacture (M) which is not an invention. We will use f to represent a known operator or idea, ϕ to represent a new operator or idea. a, b . . . will represent known variables, ways of carrying out an invention (*e.g.*, valves, chemical substances, &c.), and x, y, new variables.

It is obvious that $f(a)$ is not an invention, nor will it normally be an invention to add $f(b)$ to it. Moreover, the craftsman is not to be tied down to this. He is at perfect liberty, within limits, to make variations in his variables, to alter the size of a crank, to substitute one alkali for another, and so on; in other words, he can take $f(a+\delta a)$.

Generalising, we may say:

$$M=\Sigma f(a+\delta a).$$

Developing this by Taylor's theorem, and proceeding from an infinitesimal to a finite change, we have, neglecting quantities of the second order:

$$M=\Sigma f(a)+\Sigma\delta f(a).$$

This is the general equation for a manufacture which is not an invention. To be an invention, ingenuity (i) must be involved.

$$I=M+i \text{ or } I=\psi(M),$$

thus:

$$I=\psi[\Sigma f(a)+\Sigma\delta f(a)]=\Sigma f(a)+\Sigma\delta f(a)+i.$$

I will now consider in various actual examples the nature of ψ, the inventive function, and of i, the inventive increment.

One of the commonest cases in which a decision is necessary is that of a combination. Suppose that $f(a)$ and $f(b)$ are old; will there be invention in combining a and b?

The answer is this:

(1) $I = f(a, b)$
(2) $M = f(a) + f(b)$
(3) $I = M + i$.

If the result of the combination is given by (1), there is an invention; this is termed a "combination." If the result is given by (2), there is no invention; this is termed an "aggregation." It is interesting to compare this definition with one given by Lord Justice Buckley (Brit. United Shoe Mach. Co. *v.* Fussell) of a "combination" as "a collocation of intercommunicating parts, with a view to obtaining a simple result."

An example of a true "combination" is found in the case of Cannington *v.* Nuttall, in which a patent was upheld for a glass furnace, although each and every part (a, b, c) had been employed before in glass furnaces (employment $= f$). But, owing to the combination, and the co-operation of the parts, a new result was obtained.

$$I = f(a, b, c) = f(a) + f(b) + f(c) + i.$$

On the other hand Bridge's case is an example of an aggregation; in fact, a patent was refused by the Law Officer, showing that the case was considered absolutely devoid of invention. The alleged invention consisted in the employment in a shutter for dividing-up rooms (f) of means (a) to guide the shutters along the floor, and cogs (b) to hold the shutters against the wall. $f(a)$ and $f(b)$ were both old, and no new result flowed from their juxtaposition. Hence $M = f(a) + f(b)$: there was no invention; each part simply played its own rôle, and there was no interaction.

Another type of invention is that of varying proportions in a known combination. Here, if $M = f(a, b, c)$, and if there is a maximum at one value or range of values of c, invention may be involved. The maximum relates to the technical effect, and may be with respect to efficiency, economy, &c.

Thus if $\frac{\partial f(a, b, c_1)}{\partial c} = 0$ at the value c_1, the function will be a maximum or a minimum and there may be an invention. This will not be the case if $\frac{\partial f(a, b, c_1)}{\partial c} \neq 0$. Other singular points may be inventions, *e.g.* where $\frac{\partial f(a, b, c_1)}{\partial c} = \infty$ (discontinuity), or where $\frac{\partial^2 f(a, b, c_1)}{\partial c^2} = \infty$ (kink in the curve). This also holds for a range of values from c_1 to c_2.

Examples of the application of this equation are to be found in the cases of Edison *v.* Woodhouse, and Jandus Arc Lamp Co. *v.* Arc Lamp Co. In Edison's case f represented the employment in an incandescent lamp of an exhausted glass vessel (a), leading-in wires (b), and a carbon filament (c_1). $f(a, b, c)$ was known, but it had never been proposed to use a very thin carbon conductor or "filament." Here, owing to the high resistance and flexibility of the filament, the efficiency was a maximum:—

$\frac{\partial f(a, b, c_1)}{\partial c} = 0$, and the choice of this value c_1, which made the difference between failure and success, was held to be an invention.

In the Jandus Arc Lamp case, f represented the employment in an arc lamp of carbons (a), a tightly fitting sleeve (b), and an envelope of glass, &c. (c), inside the outer globe. By making the glass envelope 3 in. in diameter a maximum efficiency was obtained, and on this ground the patent was upheld, although envelopes had previously been made 9 in. in diameter. Here again:

$$\frac{\partial f(a, b, c_1)}{\partial c} = 0 \text{ when } c_1 = 3 \text{ in.}$$

A further example is an old case (Muntz *v.* Foster) in which a sheathing for ships was made of sixty parts of copper (a) and forty of zinc (b).

Alloys of copper and zinc had been used before in about the same proportions, but in this case the same result would not have been attained, because Muntz specified the best selected copper and highly purified zinc. The impurities (δx) were of great and unsuspected importance. Moreover, other alloys of copper and zinc (probably even of purified metals) had been made. We may consider the two points separately.

(1) Impurities:—

$f(a + \delta x, b + \delta x)$ was old, where δx represents impurities. Muntz's alloy was $f(a, b) = f(a + \delta x, b + \delta x) + i$, hence there was an invention.

(2) Selection of 60 : 40 percentage:—

$\frac{\partial f(a_{60}, b_{40})}{\partial a} = 0$ since at this percentage the efficiency was a maximum, because the alloy oxidised just fast enough to prevent barnacles adhering to the ship, but not fast enough to waste away excessively.

On the contrary, the case of Savage *v.* Harris was one in which there was held to be no invention in changing the size of part of a device for retaining ladies' hats in place. There was a back portion (a) and teeth (b), and the size of the back was altered:—

$\frac{\partial f(a_1, b)}{\partial a} \neq 0$, and there was no invention.

A known device or material (a) may be employed for a new purpose (ϕ). If $f(a)$ is the old use, and $\phi(a)$ the new use, we have for an invention $I = \phi(a) = f(a) + i$. But if $M = \phi(a) = f(a)$, there is no invention. The oft-quoted case of Harwood *v.* Great Northern Railway Company was one of the latter type. Fishplates (a) had been used for connecting (f) logs of timber, and it was held there was no invention in applying them (ϕ) to rails in which they acted in the same manner:—

$$\phi(a) = f(a).$$

But in Penn *v.* Bibby, wood (a) was employed (ϕ) for the bearings of propellers in order to allow the water to pass round the friction surfaces. Wood had previously been employed (f) in water-wheels, but $\phi(a) = f(a) + i$, and it was held that there was invention.

A similar type of invention is that in which different materials are employed in the same process. Here $f(a)$ is old, and $f(x)$ is new. If $f(x) = f(a)$ there is no invention. If $f(x) = f(a) + i$ there is invention. In the recent case, Osram Lamp Works *v.* Z Lamp Works, a patent was upheld for the use (f) in incandescent filament lamps of tungsten (x), though osmium (a) was known. Tungsten was more efficient and cheaper:—

$f(x) = f(a) + \delta i$, where δi represents a small degree of invention. This in itself might not have been sufficient, but it was coupled with the fact that one particular process of removing the carbon from the filaments was selected out of three known processes. This may be considered to require an amount of ingenuity Δi. $\delta i + \Delta i = i$, and therefore $f(x) = f(a) + i$, and there is invention involved.

Another type is the omission of one step in a known process. In the case of Badische Anilin- und Soda-Fabrik *v.* Soc. Chim. des Usines du Rhône, it was held that there was subject-matter in such an omission. A process had been proposed for preparing dyes called anisolines (A) from rhodamines (r) by first forming a potassium salt (1st step $=f$), and then transforming this salt into anisoline (2nd step $=$ F). Thus the known process was:—

$$A=F[f(r)].$$

Now it was shown that the potassium salt did not exist, *i.e.* $f(r)$ was imaginary; the patent in question obtained anisoline direct from rhodamine, $A=f(r)$, and this was held to be an invention.

I may note two final points. When a patent is granted, the criterion of ingenuity is not applied, as this is left for the Court to determine. However, if there is absolutely no ingenuity possible, the Law Officer may refuse to grant a patent. His criterion of rejection is, therefore, not $f(x)=f(a)$, as in the Court, but $f(x)\equiv f(a)$.

A patent is invalid for "insufficiency of description" if it casts on the public the burden of experiment beyond a certain point. This may be expressed by saying that in this case the equation $I=\phi(a)$ is indeterminate.

HAROLD E. POTTS.

University Club, Liverpool, April 2.

A University in the Tropics.

THE importance and value of the establishment of a university in the tropics can only be appreciated fully by those who, trained in the universities of Europe, are suddenly brought face to face with the unfamiliar conditions obtaining in a tropical country. That the proposition may be thoroughly considered and eventually realised must be the wish of all interested in the development of our tropical possessions.

The question of a site for an imperial tropical university is one upon which divergent views may be expected; few men know the equatorial belt with uniform intimacy, and are liable in consequence to be prejudiced in favour of one part or another. Admitting my own imperfect knowledge, I would like to bring forward the claims of British East Africa as an eminently suitable situation for such a university.

Dissected by the equator, it cannot be equalled for position in British territory. Rising from sea-level to plateaus more than 8000 ft. in altitude, with a mountain rising more than 17,000 ft., far above the snowline; with heavy rainfall in one part and almost rainless deserts in another; with healthy districts and parts uninhabitable by man in consequence of deadly disease; with soils varying from coral through sands to loams and clays; with standard crops from cocoanuts, rubber, and cotton, to coffee, maize, and wheat; with a large native population possessing many different languages and customs; with a flora and fauna as diversified as climate and altitude, and probably as varied as is to be found in any country; with a geological structure presenting some of the most interesting features in the world—British East Africa, the only British territory through which the equator passes, is surely uniquely situated for the seat of an imperial tropical university for the study and advancement of our knowledge of medical, agricultural, botanical, zoological, anthropological, ethnological, and other branches of science.

The capital of the country, Nairobi, is situate within 100 miles of the equator, is in a healthy district, is twenty-four hours by rail from the coast tropical belt, and the same distance from the Victoria Nyanza and Uganda, both full of the most diverse subjects of scientific interest.

The proximity of India is another great advantage in this respect. Practically all the natural conditions obtaining there—even acquaintance with the natives and their languages—may here be studied while residing in a climate resembling an English summer.

If any more suitable position for an imperial tropical university can be found than Nairobi, then the British Empire is indeed most fortunate, but a glance at the map does not suggest the possibility of such a collection of favourable factors occurring elsewhere. The passage is seventeen days, with choice of five steamship lines.

U. H. KIRKHAM.

Government Laboratory, Nairobi, February 24.

The Twinkling of Stars.

IN three papers in *The Journal of Physiology* I have described a number of new visual phenomena which show that the photochemical stimulus is situated externally to the cones, and that the foveal region is sensitised from the periphery of the retina. The result of this is that at one moment the foveal region may be the most sensitive part of the whole retina, and at another blind. The twinkling of stars may be imitated in the dark-room. If a small light be looked at in a dark-room, as, for instance, that coming through the smallest diaphragm of my colour perception lantern, which represents a 5½ in. bull's-eye railway light at a thousand yards when seen at a distance of 20 ft., care being taken not to move the eye, the light will appear to twinkle like a star. It will be noticed that pale bluish-violet circles start at the periphery of the field of vision, and, gradually contracting, reach the centre. On reaching the centre the light brightens. If the circles stop the light disappears. The colour of the circle is the same for white light or any colour.

There is another simple experiment which shows how the centre of the retina is sensitised from the periphery. On opening one eye on awaking in the morning and looking at the ceiling, the central portion is seen as an irregular, circular, rhomboidal, or star-shaped black spot. On closing the eye again a bluish-violet circle appears at the periphery or middle of the field of vision, contracts, and then, after breaking up into a star-shaped figure and becoming brighter, disappears, to be followed by another contracting circle. If the eye be opened when the star figure has formed in the centre it will appear as a bright rose-coloured star much brighter than any other part of the field of vision. If, however, we wait until the star has broken up and disappeared before opening the eye, it will be found that only a black spot is seen in the centre.

F. W. EDRIDGE-GREEN.

London, April 14.

Gain of Definition obtained by Moving a Telescope.

A SLIGHT adaptation of the explanation offered by your correspondent Mr. G. W. Butler (April 10, p. 137) appears to furnish a more natural solution of the problem. When an object at rest is seen against a background which it closely resembles there is nothing to differentiate between the object and the slight irregularities of the background. So soon as the object moves, such a differentiation becomes possible, the moving irregularities being now attributed to their real origin. It seems unnecessary to assume a "cumulative impression of contrast."

The following simple experiment lends support to this explanation. A small opening is cut in a sheet of paper covered with irregular markings, such as ink dots. Against the back of this is held another sheet similarly marked. If now the sheets are observed from such a distance that the edges of the opening are invisible, its position cannot be determined