katabolism of different species of animals is substantially proportional to their body surface. It may be surmised that the exceptional result with the hog is due to the imperfect data available for computing the body surface of this species.

[1] Armsby, *Washington, D. C., U. S. Dept. Agric., Bur. Anim. Indust., Bul.* **142,** 1912.
[2] Armsby and Fries, *Ibid., Bul.* **128,** 1911.
[3] Armsby and Fries, *J. Agric. Res., Washington,* **3,** 1915, (435); **10,** 1917, (599); **11,** 1917.
[4] Moulton, *J. Biol. Chem., New York,* **24,** 1916, (299).
[5] Benedict, Emmes, Roth and Smith, *Ibid.,* **18,** 1914, (139).
[6] Means, *Ibid.,* **21,** 1915, (263).
[7] DuBois and DuBois, *Arch. Inter. Med.,* **15,** 1915, (868).
[8] Meissl, Strohmer and Lorenz, *Zs. Biol., München* **22,** 1886, (63).
[9] Tangl, *Biochem. Zs.,* **44,** 1912, (252).
[10] Fingerling, Köhler and Reinhardt, *Landw. Versuchstat, Berlin,* **84,** 1914, (149).
[11] Zuntz and Hageman, *Landw. Jahrb., Berlin,* **27,** 1898, Ergzbd. III, (284).
[12] Voit, E., *Zs. Biol., München,* **41,** 1901, (113).

THE LOCATION OF THE SUN'S MAGNETIC AXIS

BY F. H. SEARES, A. VAN MAANEN, AND F. ELLERMAN

MOUNT WILSON SOLAR OBSERVATORY, CARNEGIE INSTITUTION OF WASHINGTON

Communicated by G. E. Hale, November 26, 1917

The discovery by Mr. Hale in 1913 of a general magnetic field[1] surrounding the sun raised at once questions of great interest. First among these was the character of the field and the variation of its intensity over the solar surface. A preliminary investigation showed that approximately the sun may be regarded as a uniformly magnetized sphere, its axis coinciding with the axis of rotation. The minuteness of the observed quantities and the difficulties experienced in their measurement necessitated the provisional acceptance of this simple hypothesis; nevertheless, well-known peculiarities of the earth's magnetic field suggested that the solar field might deviate from that of a spherical magnet, and that its axis might be inclined to the rotation axis by an amount susceptible of measurement by special series of observations. This communication is concerned with the latter of these questions, namely, the position of the magnetic axis.

Observations of the sun's field are made by placing the slit of the spectrograph in coincidence with the central solar meridian. A compound quarter-wave plate and a Nicol prism just outside the slit serve as an analyzer, the observed effect being a minute displacement of an appropriately chosen spectral line. The amount of the displacement varies with the inclination of the lines of force to the line of sight, in other words, with the position of the sun's magnetic axis, the heliographic latitude of the point observed, and the distance of the observer from the plane of the sun's equator. If the field be that of a uniformly magnetized sphere,[2]

$$k\Delta = \{ 3 \sin (2\phi - D) + \sin D \} \cos i + \{ 3 \cos (2\phi - D) + \cos D \} \sin i \cos \lambda \quad (1)$$

in which

Δ = displacement of spectral line;
ϕ = heliographic latitude of point observed;
D = angular deviation of observer from plane of sun's equator;
i = inclination of sun's magnetic axis to axis of rotation;
λ = heliographic longitude of north magnetic pole referred to central meridian;
k = constant depending on the units and the behavior of the line in a field of known intensity.

For $i = 0$, equation (1) reduces to

$$k\Delta = 3 \sin (2\phi - D) + \sin D, \quad (2)$$

and, if D also is zero, to

$$k\Delta = 3 \sin 2\phi. \quad (3)$$

Since the maximum value of D is about 7°, equation (2) differs but little from (3). The displacement curves derived by Mr. Hale from preliminary observations agreed substantially with these equations, whence it follows that i must also be small and that the difference between equations (1) and (2), which represents the influence of i, is a quantity of the second order.

When Δ is expressed in thousandths of a millimeter, k, for the lines observed, is of the order of unity. The maximum displacement, by equation (3), is therefore 3 or 4 μ (about 0.001 A). To determine the position of the magnetic axis, quantities of the order of 0.5 μ must accordingly be evaluated. This indicates sufficiently the nature of the problem and the degree of precision that had to be attained. It was evident from the beginning that a long and carefully executed series of observations would be required for a successful attack on the problem.

The original investigation by Mr. Hale was based on only four lines. Later observations have increased the number known to be affected by the sun's field to 30, for 18 of which results were communicated at the Atlanta meeting of the American Astronomical Society in December, 1913. For the investigation here described three chromium lines, $\lambda\lambda$ 5247, 5300, 5329 were selected, which are of special suitability for measurement because of intensity (2 and 3), location in the spectrum, and magnitude of displacement.

From June 8 to September 23, 1914, these lines were photographed daily under the direction of Ellerman, with almost no break in the series. The circumstances were most favorable owing to the small number of sun-spots, whose magnetic fields, many times the intensity of the underlying field of the sun, seriously complicate the investigation. Because of advantages connected with the numerical solution and the necessity of limiting what at best could be only a very laborious undertaking, the observations were confined to the zone 45°N–45°S. Twelve spectrograms with exposures of 10 to 30 minutes constituted the normal observing program for each day. For 63 of the days the photographs have been completely measured by van Maanen, who has assumed the responsibility for this part of the undertaking. More than 2000 sets of measures were required, each involving about a hundred settings of the micrometer.

In measures of minute displacements of spectral lines, systematic errors are always to be suspected, as well as the influence of prejudice arising from a knowledge of the results that will satisfy a given hypothesis. Such systematic errors as may have entered in the present case probably affect only the constant k, which varies from line to line but does not enter into the determination of the position of the magnetic axis.

To exclude the influence of prejudice, the procedure devised by Mr. Hale has been followed here. The limited zone of heliographic latitude covered by a single spectrogram may lie in the northern hemisphere, where the displacements are positive, or in the southern hemisphere, where they are negative; or it may extend over the equator and thus show only very small displacements, some negative and some positive. The measurer has rarely known in advance the latitudes covered by any spectrogram. Further, the above distribution of algebraic signs presupposes that the photograph has been made with the compound quarter-wave plate in its normal position. Since the inversion of the plate reverses the signs of the displacements, its position, as a final precaution, has been varied at random by the observer, and the measurer has not known the position used for a given photograph until after his settings were finished.

The data have been treated as follows: Each displacement affords an equation of condition of the form (1) for the determination of the unknowns k, i, and λ. The longitude of the magnetic pole, λ, involves an epoch, t_0, when the pole was on the central meridian, and the period, P, in which the magnetic axis revolves around the axis of rotation. For a single day we may assume λ to be constant, which leads us to discuss separately the observations for each day and for each line, thus deriving values for two new unknowns, x and y, which are functions of k, i, and λ. The analysis of x and y for the whole series of days then determines k, i, P, and t_0.

Equation (1) may be written

$$Ax + By = \Delta. \tag{4}$$

A and B are the bracketed expressions of (1), including only known quantities, and

$$x = k^{-1} \cos i, \qquad y = k^{-1} \sin i \cos \lambda \tag{5}$$

whence

$$Y = y/x = \tan i \cos \lambda. \tag{6}$$

About 50 values of Δ were available for each line on each day. Means were found for groups of 5 or 6 adjoining displacements, thus giving 8 or 10 observation equations of the form (4) for a least-squares determination of x and y.

The individual values of Δ for September 2, 1914, a series of average weight, are plotted in Fig. 1 against the latitudes as abscissae. The close agreement with a sine curve of the type of equation (3) appears at a glance. The calculated displacement-curves corresponding to the values of x and y derived from these data, are also shown in the figure. Their ordinates for $\phi = 0$, namely, $+0.8$, $+1.0$, and $+0.5\ \mu$, respectively, are of the order of the small quantities

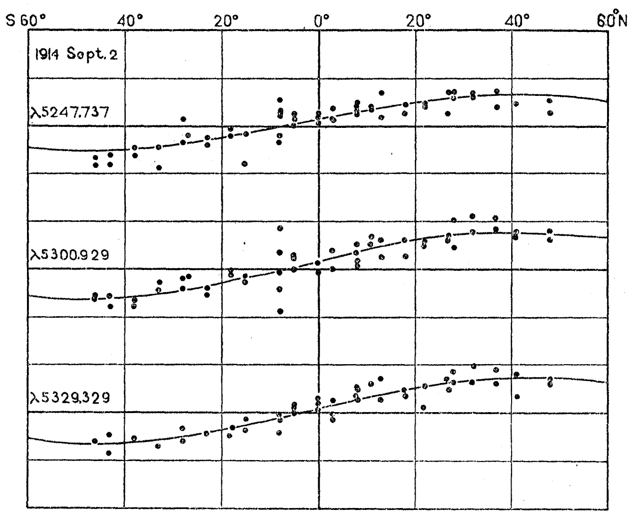

FIG. 1. DISPLACEMENT-CURVES FOR 1914, SEPTEMBER 2

Abscissae are heliographic latitudes. Ordinates are displacements, the scale being 1 division of diagram = 0.005 mm. The curves, which correspond to equation (1), have been derived from the observed values of Δ. Their ordinates for $\varphi = 0$ represent the combined influence of k, D, i, and λ. These data for the three lines give $Y = \tan i \cos \lambda = +0.213$ which is plotted as a single point in Fig. 2, together with similar values of Y for each of the other dates.

which differentiate the displacement-curve (1) from the curve (2) and indicate the precision with which the curves must be located in order to determine the value of i.

Having found x and y from each line for each day, the results were combined by (6) to form weighted mean values of Y, which were then plotted with the times as abscissae. These should define a sine curve whose amplitude and period are, respectively, $\tan i$ and P. The individual points are reproduced in Fig. 2, from which approximations for i, P and t_0 were easily derived. The final values and their probable errors

$$i = 6\overset{\circ}{.}2 \pm 0\overset{\circ}{.}4, \qquad P = 31.79 \pm 0.31 \text{ days}$$
$$t_0 = 1914, \text{ June } 25.31 \pm 0.42 \text{ days, G. M. T.}$$

were calculated by a least-squares solution giving differential corrections to the approximate values read from the curve.

The theoretical curve for tan i cos λ corresponding to these elements is also shown in Fig. 2. So far as accidental errors are concerned, the representation is very satisfactory, for the amplitude of the curve corresponds to an extreme difference of only 1 μ in the position of the individual displacement curves. There is some evidence of a gradual increase in tan i; but no emphasis is placed on the phenomenon. Whether it is real or whether it is due to errors of measurement and the influence of spots and the 'dark markings' of the Greenwich observers cannot now be determined. Days on which spots were near the central meridian were generally avoided. The magnetic fields surrounding some of the 'dark markings' noted at a later time (our attention was first directed to them in 1915) were measured and found to be two or three times as intense as the sun's general field; but their existence and influence upon the measures here discussed cannot now be traced.

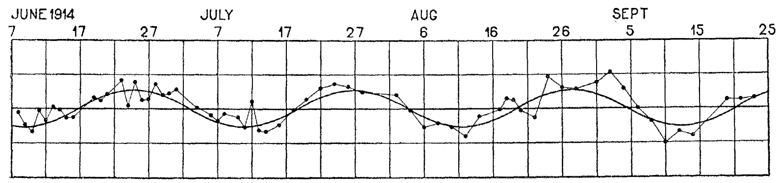

FIG. 2. THE CURVE Y = tan i cos λ

Each plotted point is derived from data for a single day similar to that illustrated in Fig. 1. Approximate values of i, P, and t_0 were read from a provisional curve. Differential corrections derived by a least-squares solution gave the final values, which correspond to the curve shown in the figure.

The result for P is of much interest in relation to the rotation period of the sun. For the reversing layer this ranges from 26.4 days (synodic period) at the equator to about 30.5 days at 45°, the value for P not being equalled until approximately 55° is reached. Hence for latitudes lower than this limit, the magnetic axis in its motion about the axis of rotation appears to lag behind the reversing layer. The bearing of this circumstance upon the physical constitution of the sun must await further elucidation. In the meantime, however, it should be noted that P may not be constant. The present series of measures, at any rate, is insufficient to establish its freedom from fluctuations; and none of the earlier observations, which were made with other purposes in view, are adapted to the investigation.

The fact that the observed values of Y agree satisfactorily with the periodic function tan i cos λ does more than show that i is not zero; it affords a most convincing demonstration of the existence of the magnetic field itself. The measures of each of the three lines, on each of the sixty odd days, define a displacement-curve which is an independent confirmation of the existence and general character of the field; but the fact that the curves for the separate days are so related to each other as to satisfy equation (6) increases enormously the weight of the conclusion and seems to exclude the possibility that it should be the consequence of an obscure and unsuspected systematic error.

We desire to express our great obligation to Miss Wolfe of the Computing Division who has executed most efficiently the numerous least-squares solutions required for the discussion of the data.

[1] Hale, G. E., *Terr. Mag., Baltimore*, **17**, 1912 (173–178); *Mt. Wilson Contr.* No. 71, *Astroph. J., Chicago*, **38**, 1913, (27–98).

[2] Seares, F. H., *Mt. Wilson Contr.* No. 72, *Astroph. J., Chicago*, **38**, 1913, (99–125).

RESONANCE AND IONIZATION POTENTIALS FOR ELECTRONS IN CADMIUM, ZINC, AND POTASSIUM VAPORS[1]

By John T. Tate and Paul D. Foote

University of Minnesota and Bureau of Standards

Communicated by R. A. Millikan, December 19, 1917

It has been shown by Franck and Hertz and others that there are, for electrons accelerated through gases or vapors, certain definite potentials at which there is a large transfer of energy from the electron to the atom, as evidenced by the emission at these potentials of radiations characteristic of the gas atom. This is to be expected on purely mechanical grounds since no considerable transfer of energy from the light electron to the relatively heavy gas atom can take place except when the time of encounter between electron and atom bears some simple relation to the characteristic period of one of the vibrational degrees of freedom in the atom. It is therefore to be expected that there will be a critical potential corresponding to each absorption line of the gas and that at this potential the electrons will give up their energy to the gas and cause the emission of a radiation of the frequency of the corresponding absorption line.

Two types of inelastic encounter between electrons and gas atoms have been observed. One of these results in the emission of a radiation of a single frequency, without ionization of the gas, while the other ionizes the gas and causes it to emit a composite spectrum of radiations. The potential giving the first type of encounter may be termed a resonance potential, that giving the second type an ionization potential.

The present paper is an account of an experimental determination of the resonance and ionization potentials in cadmium zinc, and potassium vapors.

The method employed was that described by Tate[2] for the determination of critical potentials in mercury vapor and the apparatus was similar to that used by us[3] in our work on sodium vapor.